The distribution function of the strong non-equilibrium systems of particles and anti-particles created from vacuum by electromagnetic field

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Abstract

We investigate some details of the back reaction (BR) problem formulated on the basis of the kinetic approach of authors to description of vacuum pair production in a strong electromagnetic field (Schwinger's mechanism). Here we study numerically an evolution on the distribution functions of strong non-equilibrium systems bosons and fermions. The realized analysis is shown the regular dynamics in an absence of BR mechanism is destroyed at its account. As the result some large-scale structure is arisen an a background of small-scale chaotic motions.

A Back-Reaction (BR) problem has been studied intensively in recent years in both high energy particle physics and especially in early cosmology. For simplicity, in this contribution we do not take into account the non-Abelian structure of color electric fields and concentrate on the back-reaction problem in application only to strong electromagnetic fields. However, the characteristics considered and the region of the field parameters used are of interest for the flux tube model applications to hadronic processes. We restrict ourselves to a simplest situation of a time-dependent space-homogeneous field. Under these restrictions, the peculiarity of our approach is based on making use of the exact kinetic equation (KE) [1, 2] which describes the evolution process of particle-antiparticle pair creation and annihilation in strong fields with the full zero charge.

Our approach to the BR problem is based on the following exact Vlasov-like KE for vacuum pair creation and annihilation processes in a strong time-dependent space-homogeneous electric field [1, 2] [1])

$$\frac{df(\vec{P},t)}{dt} = \frac{\partial f(\vec{P},t)}{\partial t} + eE(t)\frac{\partial f(\vec{P},t)}{\partial P_3(t)} = \mathcal{S}(\vec{P},t) \ , \tag{1}$$

[1]) We use the units $\hbar = c = 1$ and the metric is chosen to be $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$.

where $\mathcal{S}(\vec{P},t)$ is the dynamical source term describing the vacuum creation and annihilation processes within the Schwinger mechanism and neglecting the collisional damping,

$$\mathcal{S}(\vec{P},t) = \frac{1}{2}\mathcal{W}(\vec{P},t,t)\int_{-\infty}^{t} dt'\mathcal{W}(\vec{P},t,t')[1 \pm 2f(\vec{P},t')]\cos\theta(\vec{P},t,t') \ . \tag{2}$$

Here the transition amplitudes

$$\mathcal{W}(\vec{P},t,t') = eE(t')\frac{P(t,t')}{\omega^2(\vec{P},t,t')}\left(\frac{\varepsilon}{P(t,t')}\right)^{(g-1)} \ , \tag{3}$$

with the kinetic 3-vector momentum $\vec{P}$, $P(t,t') = P_3 + e[A(t) - A(t')]$ and the degeneracy factor g. We also use the notation $\vec{P}(\vec{P}_\perp, P_3)$, $\omega^2(\vec{P},t,t') = \varepsilon^2 + P^2(t,t')$ (or $\omega = \sqrt{m^2 + \vec{P}^2}$) with the transversal energy $\varepsilon = \sqrt{m^2 + P_\perp^2}$.

For the subsequent calculations it is convenient to use the following local form of the KE (1)

$$\begin{aligned}
\frac{\partial f(\vec{P},t)}{\partial t} + eE(t)\frac{\partial f(\vec{P},t)}{\partial P_3} &= \frac{1}{2}\mathcal{W}(\vec{P},t,t), v(\vec{P},t) \ , \\
\frac{\partial v(\vec{P},t)}{\partial t} + eE(t)\frac{\partial v(\vec{P},t)}{\partial P_3} &= \mathcal{W}(\vec{P},t,t)\,[1 \pm 2f(\vec{P},t)] - 2\omega\, u(\vec{P},t) \ , \\
\frac{\partial u(\vec{P},t)}{\partial t} + eE(t)\frac{\partial u(\vec{P},t)}{\partial P_3} &= 2\omega\, v(\vec{P},t) \ ,
\end{aligned} \tag{4}$$

with the initial conditions $f(t_0) = v(t_0) = u(t_0) = 0$.

The KE (1) is a direct consequence of the corresponding one-particle equation of motion in the presence of a quasi-classical electric field. In the mean-field approximation, the distribution function $f(\vec{P},t)$ allows one to find the densities of observable physical quantities. In particular, the conduction $j_{cond}(t)$ and polarization $j_{pol}(t)$ terms contribute into the electromagnetic current density [4, 6]

$$j_{in}(t) = j_{cond}(t) + j_{pol}(t) \ , \tag{5}$$

$$j_{cond}(t) = 2eg\int \frac{d^3P}{(2\pi)^3}\frac{P_3}{\omega}f(\vec{P},t) \ , \qquad j_{pol}(t) = eg\int \frac{d^3P}{(2\pi)^3}\frac{P_3}{\omega}\ v(\vec{P},t)\ \left(\frac{\varepsilon}{P}\right)^{(g-1)} \ . \tag{6}$$

The KE (1) should be combined with the Maxwell equation

$$\dot{E}(t) = -j_{tot}(t) \tag{7}$$

which closes the set of equations for the BR problem. We assume that a particle-antiparticle plasma was initially formed due to some external field $E_{ex}(t)$ excited by an external current $j_{ex}(t)$. The internal field and current are noted as $E_{in}(t)$ and $j_{in}(t)$. So, we have

$$E(t) = E_{in}(t) + E_{ex}(t) \ , \qquad j_{tot}(t) = j_{in}(t) + j_{ex}(t) \, . \tag{8}$$

It is well known that vacuum expectation values of type (6) have ultra-violet divergences and need some regularization procedure. We use the method suggested in papers [7]. To regularize different observables (currents, components of energy-momentum tensor, etc.) it is necessary to fulfill some subtractions of the relevant number of counterterms from the every regulirized function f, v and u. These subtraction terms are constructed as coefficients of the asymptotic expansion of observsble functions in series over the power of $\omega^{-1}(\vec{P})$. The leading terms of such expansions can be easily found from Eqs.(4)

$$f_a = \left(\frac{eE(t)P_3}{4\omega^3} \left(\frac{\varepsilon}{P_3} \right)^{(g-1)} \right)^2, \quad v_a = e\dot{E}(t) \frac{P_3}{4\omega^4} \left(\frac{\varepsilon}{P_3} \right)^{(g-1)} \tag{9}$$

The conduction current (6) is regular one. The polarization current (6) contains the logarithmic divergence. For its regularization it is enough to fulfill one subtraction $v \to v - v_a$ in (6) that can be interpreted as the charge renormalization. As the final result, the regularized Maxwell equation can be written in the following form (it is implied here that the coupling constant e and the fields E_{in} and E_{ex} have been already renormalized) [3]:

$$\dot{E}_{in} = -\frac{ge}{4\pi^3} \int d^3P \frac{P_3}{\omega} \left\{ f + \frac{v}{2} \left(\frac{\varepsilon}{P_3} \right)^{(g-1)} - e\dot{E}(t) \frac{P_3}{8\omega^4} \left(\frac{\varepsilon}{P_3} \right)^{2(g-1)} \right\}. \tag{10}$$

The n-wave regularization procedure is mostly adequate to the method based on reducing the KE to the system of partial differential equations (4). This approach leads to the numerical results (see [3]) which are quite consistent with the results obtained with the adiabatic regularization scheme [4]-[6].

The distribution function of a strong non-equilibrium state of particle-antiparticle plasma is investigated numerically for the cases with ($E_{in}(t) \neq 0$) and without ($E_{in}(t) = 0$) taking into account the BR mechanism. The external field is defined by the Narojny-type potential

$$A_{ex}(t) = A_0\, b\, [\tanh(t/b) + 1], \qquad E_{ex}(t) = A_0 \cosh^{-2}(t/b)\,. \tag{11}$$

The potential parameters are chosen in accordance with conditions of the flux-tube model [4]-[6]. In particular, the coupling constant is taken as a rather large value, $e^2 = 4$, dimensionless variables being used: $t \to tm, P \to P/m, A \to eA/m$. The initial impulse is characterized by the width $b = 0.5$ and amplitude $A_0 = 7.0$.

The boson and fermion distribution function are shown in Fig. 1 without the BR mechanism. A valley in the boson distribution function near small values of P_3 is due to the linear P_3-dependence of the amplitude (3). When the BR mechanism is taken into account, the regular momentum dependence of the distribution function is destroyed (Figs. 2-5; the result like that in Fig. 3 was obtained previously in [4] but on the basis of different approach). At the same time Fig. 4 and Fig. 5 demonstrate the existence of periodic temporal behavior of the distribution function. Two-dimensional representation in Fig. 5 clearly shows how the 'dog-brush' structure of the distribution function along the P_3 axis is combined with the periodic time structure along the time axis.

Thus, the BR equations generate some large-scale structure on the background of small-scale multi-mode complex dynamics. The small-scale trembling is a manifestation of vacuum oscillations. Trembling frequency is increased in the course of time. Initial smooth plot of the evolution of distribution function on Fig. 5 corresponds the action of external field impulse. Large-scale wiggle of distribution function is a consequence of the self-organization of the system [8] and stipulated by the rise of the collective plasma oscillations.

It is easy to express an assumption about stochastic behavior of the system provided that the BR mechanism is included in consideration. It is possible that a dynamical chaos can be found at other values of parameter eE. However, the rigorous verification of this assumption needs the special numerical investigation. It is quite possible that uncovered irregularity in the vacuum pair production dynamics is one of the statistic rules sources, observed in multiple particle creation processes at relativistic ions collisions.

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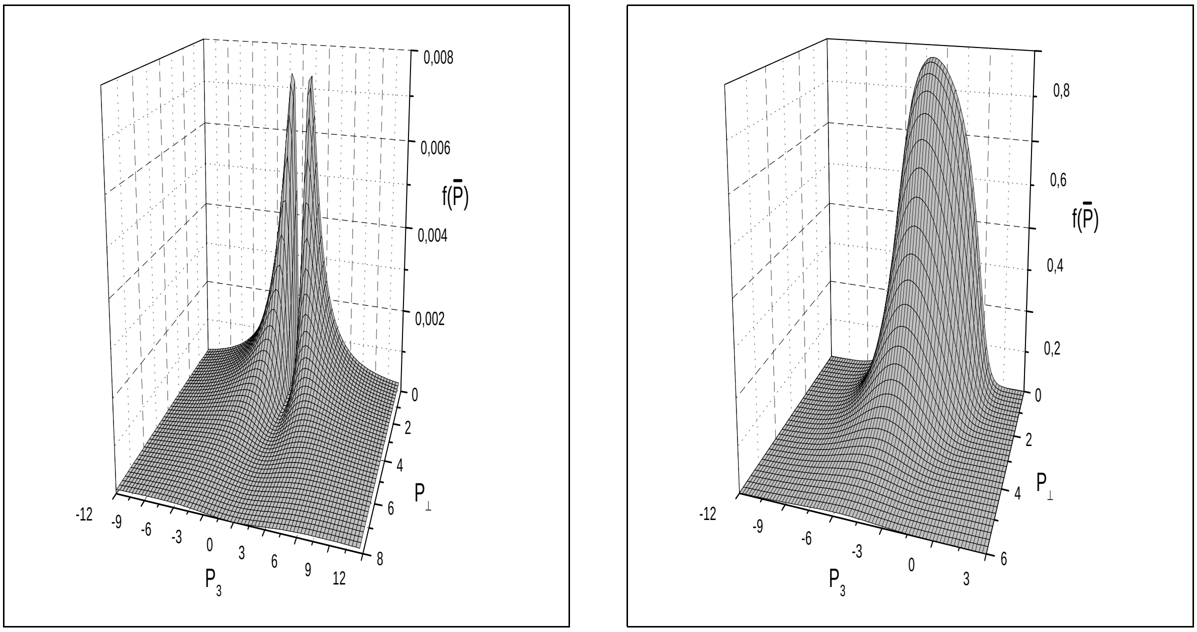

Figure 1: Momentum distribution of produced bosons (left) and fermions (right) at time $t = 0.05$. The influence of the back reaction is not included.

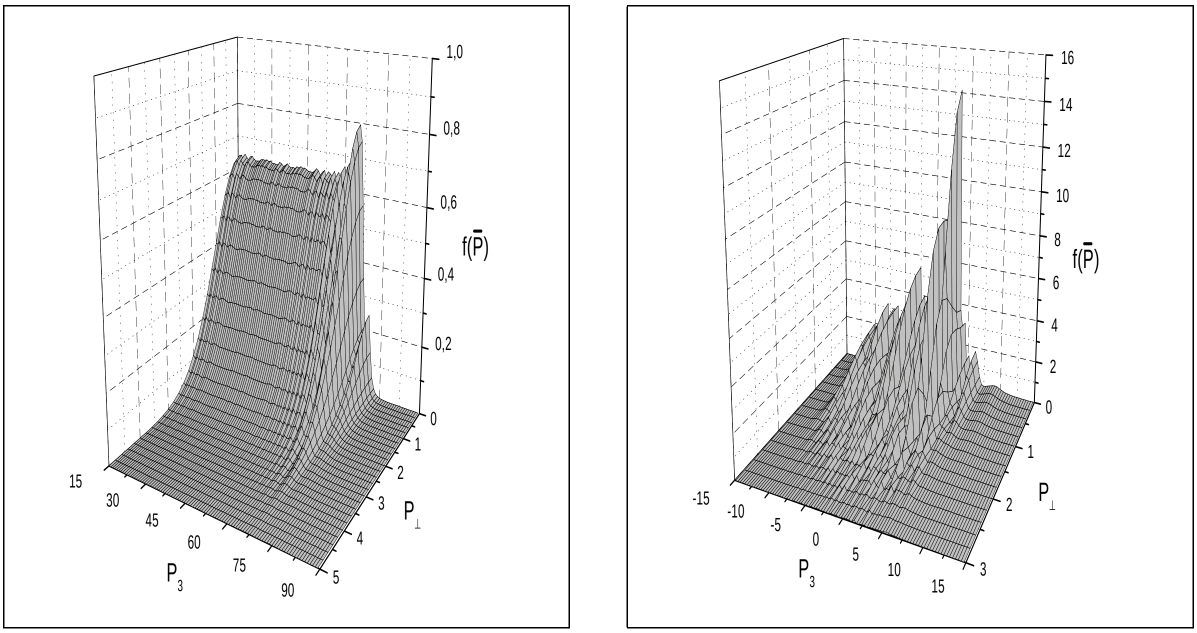

Figure 2: Momentum distributions of produced bosons without (left) and with inclusion (right) of the back reaction at time $t = 10$.

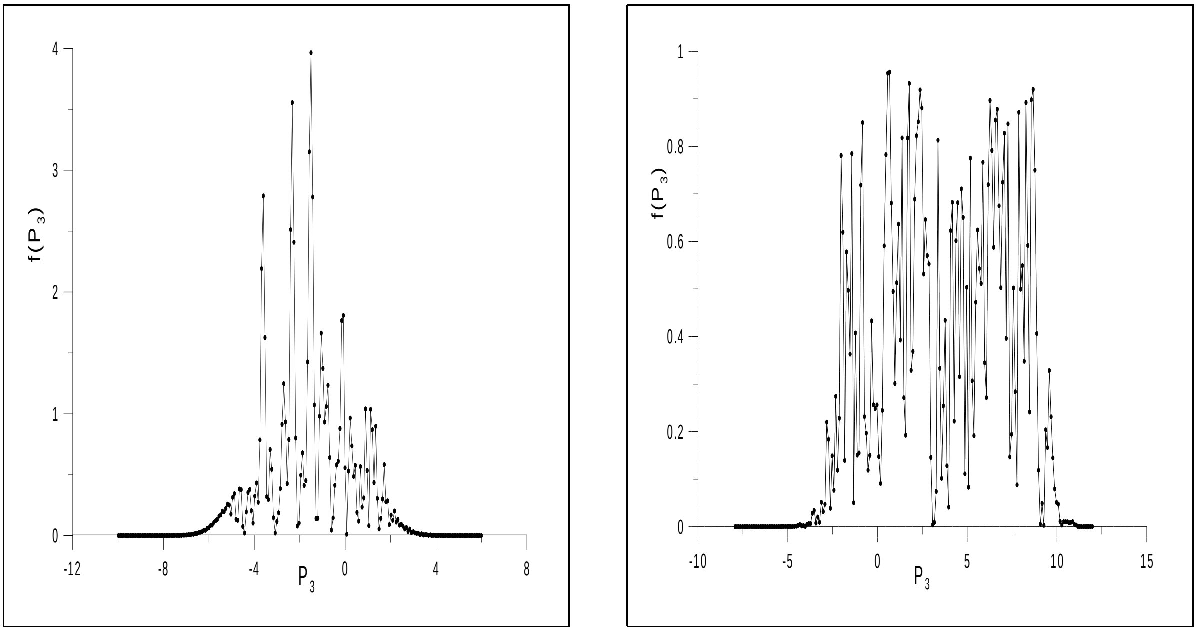

Figure 3: Momentum distributions of produced bosons (left) and fermions (right) at time $t = 25$. Transversal momentum $P_\perp = 0$ in both cases.

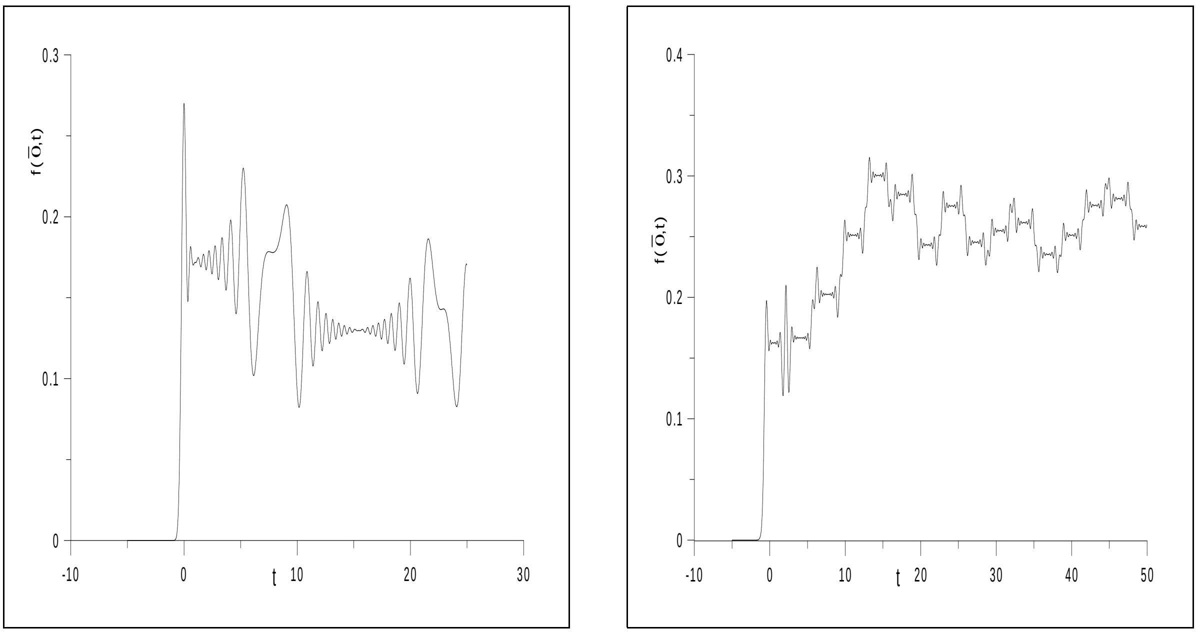

Figure 4: Time evolution of boson (left) and fermion (right) distribution function $f(\vec{0}, t)$.

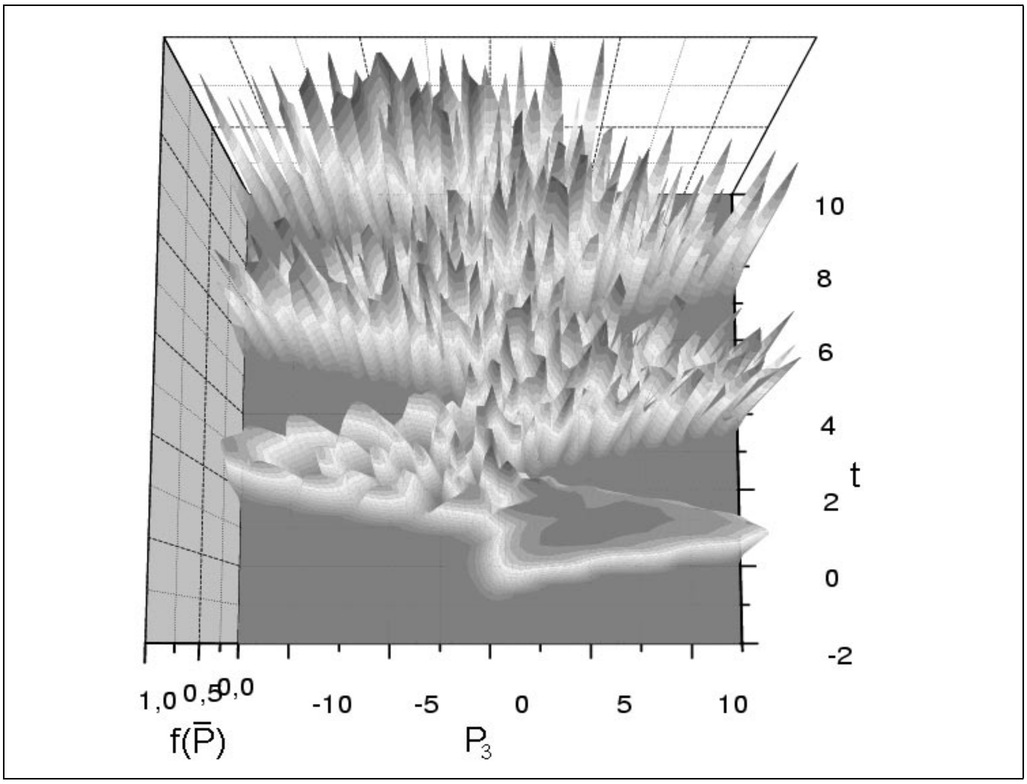

Figure 5: Space-time evolution of the fermion distribution function. Transversal momentum $P_\perp = 0$.